Acoustic diffraction effects at the Hellenistic amphitheater of Epidaurus: Seat rows responsible for the marvelous acoustics

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The Hellenistic theater of Epidaurus, on the Peloponnese in Greece, attracts thousands of visitors every year who are all amazed by the fact that sound coming from the middle of the theater reaches the outer seats, apparently without too much loss of intensity. The theater, renowned for its extraordinary acoustics, is one of the best conserved of its kind in the world. It was used for musical and poetical contests and theatrical performances. The presented numerical study reveals that the seat rows of the theater, unexpectedly play an essential role in the acoustics—at least when the theater is not fully filled with spectators. The seats, which constitute a corrugated surface, serve as an acoustic filter that passes sound coming from the stage at the expense of surrounding acoustic noise. Whether a coincidence or not, the theater of Epidaurus was built with optimized shape and dimensions. Understanding and application of corrugated surfaces as filters rather than merely as diffuse scatterers of sound, may become imperative in the future design of modern theaters. [DOI: 10.1121/1.2709842]

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I. INTRODUCTION

In the classical world, the "asclepieion" at Epidaurus was the most celebrated and prosperous healing center;[1] in its vicinity there was the amphitheater, designed by Polycleitus the Younger in the fourth century B.C. and famous for its beauty and symmetry. The original 34 seat rows were extended in Roman times by another 21 rows. The theater is well preserved because it has been covered for centuries by thick layers of earth. A recent picture of the theater is presented in Fig. 1.

Marcus Vitruvius Pollio (first century B.C.) describes in his famous books "De Architectura"[2] the state of the art in architecture and shows evidence that man was aware of the physical existence of sound waves. He writes, "Therefore the ancient architects following nature's footsteps, traced the voice as it rose, and carried out the ascent of the theater seats. By the rules of mathematics and the method of music, they sought to make the voices from the stage rise more clearly and sweetly to the spectators' ears. For just as organs which have bronze plates or horn sounding boards are brought to the clear sound of string instruments, so by the arrangement of theaters in accordance with the science of harmony, the ancients increased the power of the voice."

This indicates that the construction of theaters was performed according to experimental knowledge and experience and that it was done such as to improve the transmission of sound from the center of the theater (the orchestra) toward the outer seats of the "cavea." It has however always been believed, even in the same chapter written by Vitruvius,[2] or the work by Izenour,[3] that it was mainly the aspect of the slope of the theater, as a result of the constructed seats, rather than the seats themselves, that have been a key factor in the resulting acoustic properties.

The current study was triggered by the marvels of Epidaurus and by recent advances in the explanation of a variety of diffraction effects on corrugated surfaces.[4–9]

The theory of diffraction of sound is based on the concepts of the Rayleigh decomposition of the reflected and transmitted sound fields.[10–12] The theory, earlier applied to describe a number of diffraction effects for normal incident ultrasound on corrugated surfaces,[13] has been used successfully to understand the generation of ultrasonic surface waves in the framework of nondestructive testing. The theory was later expanded to include inhomogeneous waves and enabled a description and understanding of the backward displacement of bounded ultrasonic beams obliquely incident on corrugated surfaces, a phenomenon which had been obscure for 3 decades.[9,14] Even more, it was later exposed that predictions resulting from that theory were in perfect agreement with new experiments.[15]

An expansion of the theory to pulsed spherical acoustic waves revealed special acoustic effects at Chichen Itza in Mexico.[7,8] The advantage of the theory is its ability to make quantitative simulations as they appear in reality. From those simulations, it is then possible to detect and characterize patterns and characteristics of the diffracted sound field such as in the case of a short sound pulse incident on the staircase of the El Castillo pyramid in Chichen Itza. The study indicated that the effects were slightly more complicated than the earlier considered principle of Bragg scattering. In the meantime, the fact that the Quetzal echo at Chichen Itza is influ-

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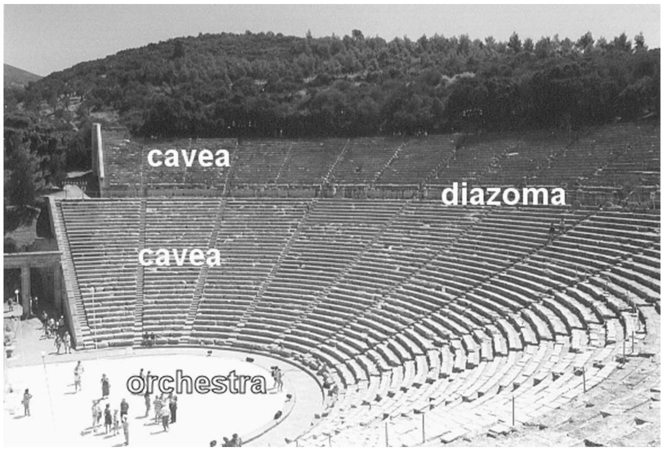

FIG. 1. Picture of the theater of Epidaurus (picture taken by the authors).

enced by the properties of the sound source as well as the existence of the "raindrop effect," have been experimentally verified by Cruz *et al.*[16] Bilsen[17] later showed that if one is only interested in the position of time delay lines on a sonogram and not in the entire amplitude pattern, that it is possible to apply a simpler model based on the gliding pitch theory.

For a study of acoustic effects at Epidaurus however, we are not interested in the response to a pulse. We are merely interested in how, for each frequency, sound behaves after interaction with the seats of the theater. Therefore the extensive diffraction theory, as used earlier,[7] is the pre-eminent tool.

Until now, there have appeared a number of "explanations" for the excellent acoustics of Epidaurus, such as that sound is driven by the wind because the wind is mostly directed from the orchestra toward the cavea. The wind direction has indeed some influence, but it is also known that the acoustics of Epidaurus is very good when there is no wind or when wind comes from other directions; wind even has a general negative effect because it produces undesirable noise. Another theory is the importance of the rhythm of speech but there are also modern performances taking place at Epidaurus where the typical rhythm of Hellenistic poems and performances composed by Homerus, Aeschylus, Sophocles, or Euripides is not there; still the acoustics seems perfect.

The last theory is that special masks, worn by performers, may have had a focusing effect on the generated sound, but that does not explain why speakers with weak voices are also heard throughout the theater.

Izenour[3] points out that the acoustics is so good because of the clear path between the speaker and the audience. The current work proves numerically that the effect of diffraction on the seat rows is probably an even more important effect than the "clear path effect."

In what follows, we describe the geometry of the theater. Consequently we explain briefly how the numerical simulations are performed. Then we present and explain the numerical results. We end our paper with the most important conclusions.

The material parameters at Epidaurus have been taken as: 2000 kg/m^3 for the density of the theater's limestone, and a shear wave velocity of 2300 m/s and longitudinal wave velocity of 4100 m/s.

For the air at Epidaurus, we have taken two cases: "summer," corresponding to an air density of 1.172 kg/m^3 and a (longitudinal) wave velocity of 348.04 m/s; and "winter," corresponding to an air density of 1.247 kg/m^3 and a (longitudinal) wave velocity of 337.50 m/s.

II. GEOMETRY: MILLER PROJECTION

In this paper, we only focus on the geometrical properties of the theater that are important for the acoustics. The theater is almost semicircular. This means that the acoustics, for a sound source situated at the center of the theater, will have a circular symmetry similar to the theater itself. A Miller projection (as in cartography), mathematically transforming the semicircular theater into a rectangular theater resulting in seat rows in the cavea becoming straight rows having the same length as the outer seat row; and transforming or "stretching" the central spot (at the center of the orchestra) into a straight line parallel with the transformed theater and having the same length as the seat rows; makes the sound source at the center of the orchestra become a line source that generates cylindrical waves. For simplicity, we do not take into account edge effects at the edges of the seat rows. We may therefore disregard one Cartesian coordinate and study the entire problem in a two-dimensional space.

All this is physically correct if we also perform a Miller projection of the entire sound field. In other words the sound amplitude must be multiplied by a function describing the sound density variation along the theater slope due to the Miller projection. An inverse function must then be applied to the results if we want to transform the results back to the circular theater. Conveniently it is therefore unnecessary to consider this function because we do not want to show results that are valid for the transformed theater, but in the real circular theater. Furthermore a source not exactly situated at the center of the orchestra will deliver exact results along the theater radius passing through the source but will yield slightly deviating results for other positions in the theater's cavea.

III. GEOMETRY: SHAPE, SIZE, AND DISTANCES

A relict of the extension from 34 seat row to 55 in Roman times is the presence of a "diazoma" in between both constructions as can be seen in Fig. 1. This acoustic discontinuity is neglected in our study because the upper seat rows are built along the same straight line as the inner seat rows and this neglect mathematically corresponds to adding a few seat rows and makes the two-piece theater a single-piece theater having 60 seat rows instead of 55.

It is necessary to define a number of vectors and angles of importance. They are depicted in Figs. 2 and 3.

Tables I–III explain the variables depicted in Figs. 2 and 3 and indicate the numerical values for the theater at Epidaurus.

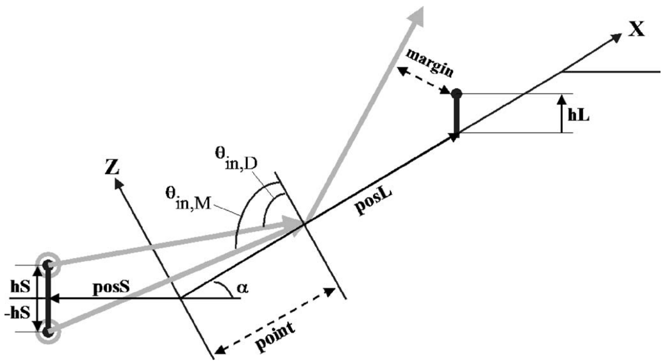

FIG. 2. Vectors and angles used to describe the theater and the acoustics.

Straightforward geometrical considerations yield for the direct distance between source and receiver:

$$\mathrm{RD} = \sqrt{(\mathrm{posL}\cos(\alpha)+\mathrm{posS})^2+(\mathrm{hL}+\mathrm{posL}\sin(\alpha)-\mathrm{hS})^2}. \tag{1}$$

Analogously we obtain for the distance between source and receiver, taking into account the mirror effect caused by the foreground:

$$\mathrm{RM} = \sqrt{(\mathrm{posS}+\mathrm{posL}\cos(\alpha))^2+(\mathrm{hL}+\mathrm{posL}\sin(\alpha)+\mathrm{hS})^2}. \tag{2}$$

The numerical procedure developed for this paper is based on consecutive consideration of diffraction in subsequent spots of diffraction. With respect to these spots "P" of diffraction, we define a number of valuable distances:

$$\mathrm{RSP} = \sqrt{(P-\mathrm{hS}x-\mathrm{posS}x)^2+(\mathrm{posS}z+\mathrm{hS}z)^2} \tag{3}$$

which is the distance between the actual sound source and the spot of diffraction;

$$\mathrm{RMP} = \sqrt{(P+\mathrm{hS}x-\mathrm{posS}x)^2+(\mathrm{posS}z-\mathrm{hS}z)^2} \tag{4}$$

is the distance between the mirror source and the spot of diffraction and

$$\mathrm{RPL} = \sqrt{(xL-P)^2+zL^2}, \tag{5}$$

being the distance between the diffraction spot and the listener where

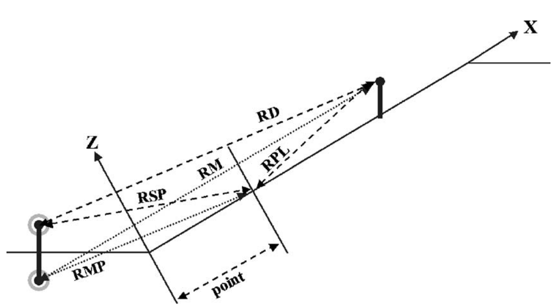

FIG. 3. Vectors and angles used to describe the theater and the acoustics.

TABLE I. Measured values describing the theater.[a]

Quantity	Value	Meaning
b	0.746 m	Width of the seats
α	26.6°	Slope of the theater
$\mathrm{SD}_{\mathrm{max}}$	22.63 m	Distance between center of orchestra and lower seat row
l_{theater}	49.88 m	Length of the seats

[a]See Ref. 1.

$$xL = \mathrm{posL}+\mathrm{hL}\sin(\alpha) \quad \text{and} \quad zL = \mathrm{hL}\cos(\alpha). \tag{6}$$

A considered ray of sound is incident at the diffraction spot at the angle of incidence θ_{in} (cf. Fig. 2). If we do not consider any reflections on the foreground, then this angle is equal to

$$\theta_{\mathrm{in},D} = \arccos\left(\frac{\mathrm{posS}z+\mathrm{hS}z}{\mathrm{RSP}}\right). \tag{7}$$

If we also consider a reflection on the foreground then the angle of incidence is

$$\theta_{\mathrm{in},M} = \arccos\left(\frac{\mathrm{posS}z-\mathrm{hS}z}{\mathrm{RMP}}\right). \tag{8}$$

IV. ACOUSTIC SIMULATIONS

A. Sound field description

As noted earlier, the sound source is considered cylindrical. The generated sound field is thought of as a bunch of rays spread over all directions and widening with increased distance (involving an amplitude inversely proportional to the square root of the traveled distance) from the source just as a real cylindrical sound field. The phase of the sound within each considered beam also behaves as the actual cylindrical sound field. Normalization yields the summation of the amplitudes of all beams to be equal to unity. These rays interact with the theater. Because at considerable distances from the sound source the sound field pattern in each of the "rays" approximates a plane wave, the interaction of the rays with the theater is modeled as a plane wave interaction, allowing the use of earlier developed techniques based upon Rayleigh's theory of diffraction.[9,10,13] The diffracted sound fields are also thought of as a widening sound ray continuing the same widening pattern and pace as the incident sound ray. We apply Rayleigh's decomposition, therefore the incident sound field (displacement field) is given by

$$\mathbf{N}^{\mathrm{inc}} = A^{\mathrm{inc}}\varphi^{\mathrm{inc}}(ik_x^{\mathrm{inc}}\mathbf{e}_x+ik_z^{\mathrm{inc}}\mathbf{e}_z). \tag{9}$$

The reflected ($\zeta=r$) and transmitted longitudinal ($\zeta=d$) sound fields are given by

TABLE II. Calculated values describing the theater.

Quantity	Equal to	Value	Meaning
h	$b\tan(\alpha)$	0.367 m	Height of the seats
Λ	$\sqrt{b^2+h^2}$	0.831 m	Periodicity of the seat rows
n	$\frac{l_{\mathrm{theater}}}{\Lambda}$	60	Number of seat rows

TABLE III. Explanation of abbreviations used throughout the paper.

Abbreviation	Explanation
hS	height of the source
posS	position of the source
hL	height of the listener
posL	position of the listener
RD	direct distance
RSP	distance between source and point of diffraction
RPL	distance between point of diffraction and listener
RM	direct distance for mirror source
RMP	distance between mirror source and diffraction point

$$\mathrm{N}^{\varsigma} = \sum_m A_m^{\varsigma} \varphi^{m,\varsigma}(ik_x^{m,\varsigma} e_x + ik_z^{m,\varsigma} e_z), \quad \varsigma = r, d. \tag{10}$$

Finally, the transmitted shear sound field is written as

$$\mathbf{N}^s = \sum_m A_m^s \mathbf{P}^{m,s} \varphi^{m,s} \tag{11}$$

with

$$\varphi^{\tau} = \exp i(k_x^{\tau} x + k_z^{\tau} z) \tag{12}$$

and

$$k_x^{m,s} P_x^{m,s} + k_z^{m,s} P_z^{m,s} = 0. \tag{13}$$

B. Mechanical continuity conditions

The sound fields described in Eqs. (9) and (10) must correspond to incident and diffracted sound on the air-solid interface formed by the seat rows.

In order to determine the unknown coefficients A_m^r, A_m^d, $A_m^s P_x^{m,s}$, and $A_m^s P_z^{m,s}$ we impose continuity of normal stress and normal displacement everywhere along the interface between air and solid. The corrugated surface is given by a function $z=f(x)$. Periodicity of the corrugation yields

$$f(x+\Lambda) = f(x) \tag{14}$$

with Λ the corrugation period. For further use, we define the function $g(x,z)$ as follows:

$$g(x,z) = f(x) - z. \tag{15}$$

Along the interface we have $g(x,z)=0$.

We do not consider viscous damping effects. The stress tensor T^{τ} ($\tau=1$ in air, $\tau=2$ the solid), is calculated as

$$T_{ij}^{\zeta} = \sum_{\eta} \lambda^{\tau} \varepsilon_{\eta\eta}^{\tau} \delta_{i,j} + 2\mu^{\tau} \varepsilon_{i,j}^{\tau} \tag{16}$$

in which λ^{τ} and μ^{τ} are Lamé's constants.

The strain tensor ε^{τ} is calculated as

$$\varepsilon_{ij}^{\tau} = \tfrac{1}{2}(\partial_i N_j^{\tau} + \partial_j N_i^{\tau}). \tag{17}$$

We also incorporate the dispersion relations for longitudinal waves

$$k^{\zeta} = \sqrt{\frac{\rho\omega^2}{\lambda^{\tau} + 2\mu^{\tau}}} \tag{18}$$

with ζ="inc" or "m,r" and for shear waves

$$k^{\zeta} = \sqrt{\frac{\rho\omega^2}{\mu^{\tau}}} \tag{19}$$

with $\zeta = s, 2$ for shear waves in the solid.

The dispersion relations (18) and (19) reveal the value of k_z corresponding to each of the values for k_x for the different diffraction orders. The sign of k_z is chosen according to the well-known "Sommerfeld conditions" stating that each of the generated waves must propagate away from the interface and demanding that whenever k_z is purely imaginary (evanescent waves), its sign must be chosen such that the amplitude of the wave under consideration diminishes away from the interface.

Continuity of normal stress and normal displacement everywhere along the interface between air and solid yield

$$(\mathbf{N}^{\text{inc}} + \mathbf{N}^r) \cdot \boldsymbol{\nabla} g = (\mathbf{N}^d + \mathbf{N}^s) \cdot \boldsymbol{\nabla} g \quad \text{along } g = 0, \tag{20}$$

$$\sum_j T_{ij}^1 (\nabla g)_j = \sum_j T_{ij}^2 (\boldsymbol{\nabla} g)_j \quad \text{along } g = 0. \tag{21}$$

Relations (13), (20), and (21) result in four equations that are periodical along the x axis. A discrete Fourier transform with repetition period Λ is eminent and each of the Fourier components on both sides of the equations are then equal to one another.

Straightforward calculations ultimately result in four continuity equations

Equation 1:

$$\begin{aligned} & A^{\text{inc}} I^{\text{inc},p} i(-(k^1)^2 + k_x^{\text{inc}} k_x^p) + \sum_m A_m^r I^{m,r,p} i(-(k^1)^2 + k_x^m k_x^p) \\ & + \sum_m A_m^d I^{m,d,p} i(-(k^{d,2})^2 + k_x^m k_x^p) - \sum_m A_m^s P_x^{m,s} I^{m,s,p} \\ & \times (k_x^p - k_x^m) + \sum_m A_m^s P_z^{m,s} I^{m,s,p} (k_z^{m,s}) = 0. \end{aligned} \tag{22}$$

Equation 2:

$$\begin{aligned} & -A^{\text{inc}} I^{\text{inc},p} \rho_1 (k_x^p - k_x^{\text{inc}}) - \sum_m A_m^r I^{m,r,p} \rho_1 (k_x^p - k_x^m) \\ & + \sum_m A_m^d I^{m,d,p} \rho_2 \left(-k_x^m + \left(1 + 2\frac{(k_x^m)^2 - (k^{d,2})^2}{(k^{s,2})^2} \right) k_x^p \right) \\ & + \sum_m A_m^s P_x^{m,s} I^{m,s,p} i\rho_2 \left(1 - \frac{k_x^m k_x^p}{(k^{d,2})^2} + \left(\frac{1}{(k^{d,2})^2} \right.\right. \\ & \left.\left. - \frac{1}{(k^{s,2})^2} \right) (k_x^m)^2 \right) + \sum_m A_m^s P_z^{m,s} I^{m,s,p} i\rho_2 (k_z^{m,s}) \left(\left(\frac{1}{(k^{d,2})^2} \right.\right. \\ & \left.\left. - \frac{1}{(k^{s,2})^2} \right) k_x^m - \left(\frac{1}{(k^{d,2})^2} - \frac{2}{(k^{s,2})^2} \right) k_x^p \right) = 0. \end{aligned} \tag{23}$$

Equation 3:

$$A^{\mathrm{inc}} I^{\mathrm{inc},p} \rho_1 (k_z^{\mathrm{inc}}) + \sum_m A_m^r I^{m,r,p} \rho_1 (k_z^{m,r}) + \sum_m A_m^d I^{m,d,p} (k_z^{m,d}) \rho_2 \left(-1 + \frac{2}{(k^{s,2})^2} (k_x^m k_x^p) \right) + \sum_m A_m^s P_x^{m,s} I^{m,s,p} i (k_z^{m,s}) \rho_2 \left(\left(\frac{1}{(k^{d,2})^2} - \frac{1}{(k^{s,2})^2} \right) k_x^m - \frac{k_x^p}{(k^{s,2})^2} \right) + \sum_m A_m^s P_z^{m,s} I^{m,s,p} i \rho_2 \left(\left(\frac{1}{(k^{d,2})^2} - \frac{1}{(k^{s,2})^2} \right) \times (k_z^{m,s})^2 + 1 - \frac{k_x^m k_x^p}{(k^{s,2})^2} \right) = 0. \quad (24)$$

Equation 4:

$$(A_m^s P_x^{m,s} k_x^{m,s} + A_m^s P_z^{m,s} k_z^{m,s}) \delta_{m,p} = 0. \quad (25)$$

$\delta_{m,p}$ in Eq. (25) is Kronecker's delta.

The grating equation (similar to the one in optics) takes care of k_x^m and k_x^p as follows:

$$k_x^\beta = k_x^{\mathrm{inc}} + \beta \frac{2\pi}{\Lambda}, \quad \beta = m,p \in \mathbb{Z}. \quad (26)$$

The Fourier transformation also leaves integrals within Eqs. (22)–(24):

$$I^{\mathrm{inc},\eta} = \frac{1}{k_z^{\mathrm{inc}}} \int_\Lambda \exp i[(k_x^{\mathrm{inc}} - k_x^\eta) x + k_z^{\mathrm{inc}} f(x)] dx, \quad (27)$$

$$I^{m,\xi,\eta} = \frac{1}{k_z^{m,\xi}} \int_\Lambda \exp i[(k_x^m - k_x^\eta) x + k_z^{m,\xi} f(x)] dx. \quad (28)$$

The integrals (27) and (28) can be solved numerically or analytically.[9] They contain information about the surface and are therefore called "surface integrals."

C. The number of diffraction orders

Equations (22)–(25) actually represent an infinite number of equations and unknown variables because the orders m and p constitute a discrete infinite interval of integer numbers $\mathbb{Z}$. As discussed in earlier papers,[5,7,9,13] finiteness of energy makes a limitation of the number of diffraction orders authorized because only a few orders are really propagating; the others are evanescent and with increasing value of m or p, play a less important role in the energy transformation upon diffraction.[13] Therefore we only take into account two forward and two backward "propagating" evanescent waves for each of the considered frequencies. In other words, for each frequency we consider the propagating bulk waves (their number depends on the frequency) and add two more evanescent waves in each direction. The developed procedure therefore automatically determines the number of waves involved.

D. The formation of observed diffracted sound per generated ray

By "observed diffracted sound" we mean sound that reaches a given observer in the cavea of the theater. Because we model the acoustics by means of cylindrically expanding rays, it is clear that not all of these diffracted rays will ultimately reach the observer.

It is known from textbooks on geometry that the distance from the diffracted ray to the observer is given by

$$\left| \frac{(xL + \mathrm{pos}L - p) \dfrac{\mathrm{Re}(k_z^m)}{\mathrm{Re}(k_x^m)} - zL}{\sqrt{\left(\dfrac{\mathrm{Re}(k_z^m)}{\mathrm{Re}(k_x^m)} \right)^2 - 1}} \right|. \quad (29)$$

If this distance is smaller than a predetermined limit, the ray is considered to reach the observer. If the distance is larger, we further discard that ray. The limit is determined by the width of the ray at the point of observance. We approximate this width by its slightly larger value

$$\mathrm{limit} = 2 \max(\theta_{\mathrm{in}}^{+1}, \theta_{\mathrm{in}}^{-1})(\mathrm{RSP} + \mathrm{RPL}) \quad (30)$$

with $\theta_{\mathrm{in}}^{+1}$ and $\theta_{\mathrm{in}}^{-1}$ the angle (in rad) between the considered ray and the consecutive ray, respectively, the angle between the considered ray and the preceding ray.

Whenever we consider sound that is reflected in the orchestra on the foreground of the theater, we replace "RSP" by "RMP" in Eq. (30).

E. The integrated effect for all considered rays

The previous paragraph describes the interaction of one ray at one single spot of the theater and it is determined whether or not an observer will "detect" or "hear" the diffracted rays. Calculation of the integrated effect consists of a repetition of the previous procedures for each considered generated ray and adding up all rays that are detected by the observer. To approach physical reality, we model the generated cylindrical sound field by a bunch of rays that fulfill specific incorporated requirements. The distribution of rays is made such that the rays would be incident at spots P on the theater at equally spaced positions and such that there would be three spots of incidence per wavelength, therefore producing realistic simulations. Furthermore, we can apply the above-mentioned procedure for each position of the listener "posL" and then plot the result as a function of the position of the listeners on the theater.

F. The Lipmann and Wirgin criteria

The considered model, based on Rayleigh's decomposition, is a simplified approach of more complicated models such as the differential[18,19] and integral equation approach[20–23] and Waterman's theory;[24–28] it is not valid for any situation. There are certain requirements that need to be fulfilled as studied by Lipmann[29] and later also by Wirgin.[12]

Wirgin has shown[12] that "contrary to prevailing opinion, the Rayleigh theory is fully capable of describing the scattering phenomena produced by a wide class of corrugated

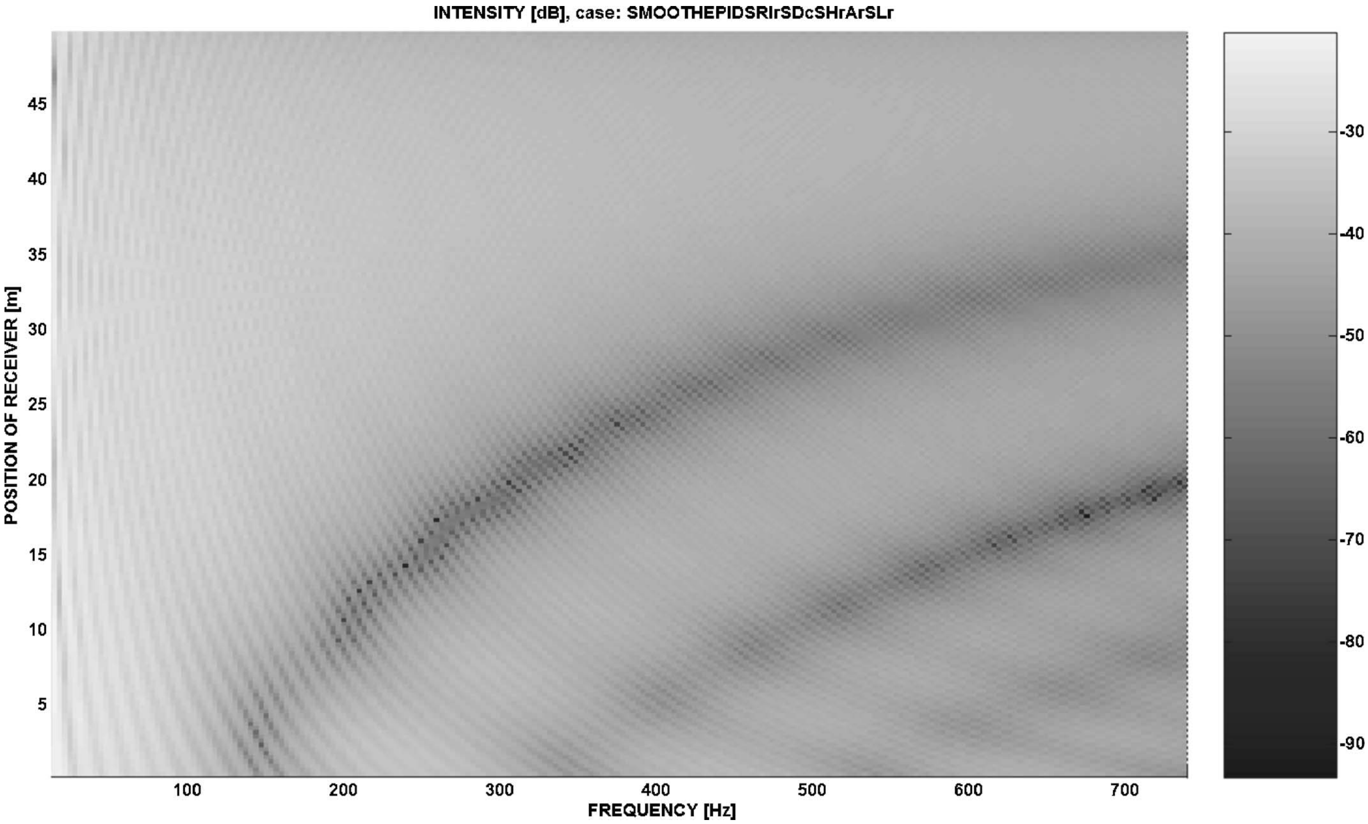

FIG. 4. The received intensity in decibels for listeners situated at heights along the slope of a smooth theater, i.e., without seat rows, given along the vertical axis and for frequencies given along the horizontal axis. The geometry corresponds to the geometry of Epidaurus and the sound source is situated at 22.63 m from the first row of seats, i.e., at the center of the theater. Reflections on the foreground are neglected.

surfaces, including those whose roughness is rather large." Furthermore, Wirgin[12] proves that the Rayleigh theory is valid for λ the largest wavelength involved in the diffraction phenomenon, for Λ the corrugation period and for h the corrugation height, whenever

$$h < 0.34\Lambda \qquad (31)$$

and

$$\lambda > 1.53348h. \qquad (32)$$

The Wirgin criteria are somewhat tighter than the older Lipmann criteria.[22,29] Nevertheless we may expect that the Rayleigh theory for our purpose is reliable for frequencies below 750 Hz (in summer and in winter). Actually the theory may even be valid to a large extent above 800 Hz. A limitation to 750 Hz means that for a piano with 88 keys, our model would simulate the acoustics at Epidaurus for the first 58 keys, this is almost 70% and is not too bad.

V. NUMERICAL RESULTS

In all our calculations we have considered an observer whose ears are 80 cm above his seat. First consider a smooth Epidaurus theater, i.e., a theater that consists of a smooth slope without seat rows, making the sound rays undergo no diffraction but simple reflections on the slope transmitting a part of their energy into the limestone slope and reflecting most of their energy. Calculations then reveal the observed frequency spectrum for all positions posL on the slope. Consider a sound source placed in the center at 22.63 m from the first seat row and having a height of 2 m. This height is reasonable since in the Hellenistic era the performers, who were not very tall, wore "cothurns" or high theater sandals.

For simplicity, reflections on the foreground are not considered for the moment. Figure 4 shows the calculated results. The grayscale indicates the received sound intensity whereas the horizontal axis gives the frequency and the vertical axis corresponds to the height along the slope of the theater (posL).

Notice the appearance of distinct patterns due to the interference between sound reaching the listener uninterrupted and sound reaching the listener after being reflected upon the slope of the theater. The "bands" of diminished intensity are actually due to a phase canceling effect and are positions where the audience will receive a much lower intensity than at other positions in the cavea.

Figure 5 is similar to Fig. 4, except that here reflections on the foreground are also taken into account, resulting in a more complicated pattern, but with less distinct regions of diminished intensity. Reflections on the foreground are therefore responsible for a better distribution of sound throughout the theater.

Consider the situation at Epidaurus. Figure 6 shows the results for Epidaurus with the seat rows installed (at a periodicity of 0,831 m) and with reflections on the foreground. The sound source is again situated as in Fig. 5.

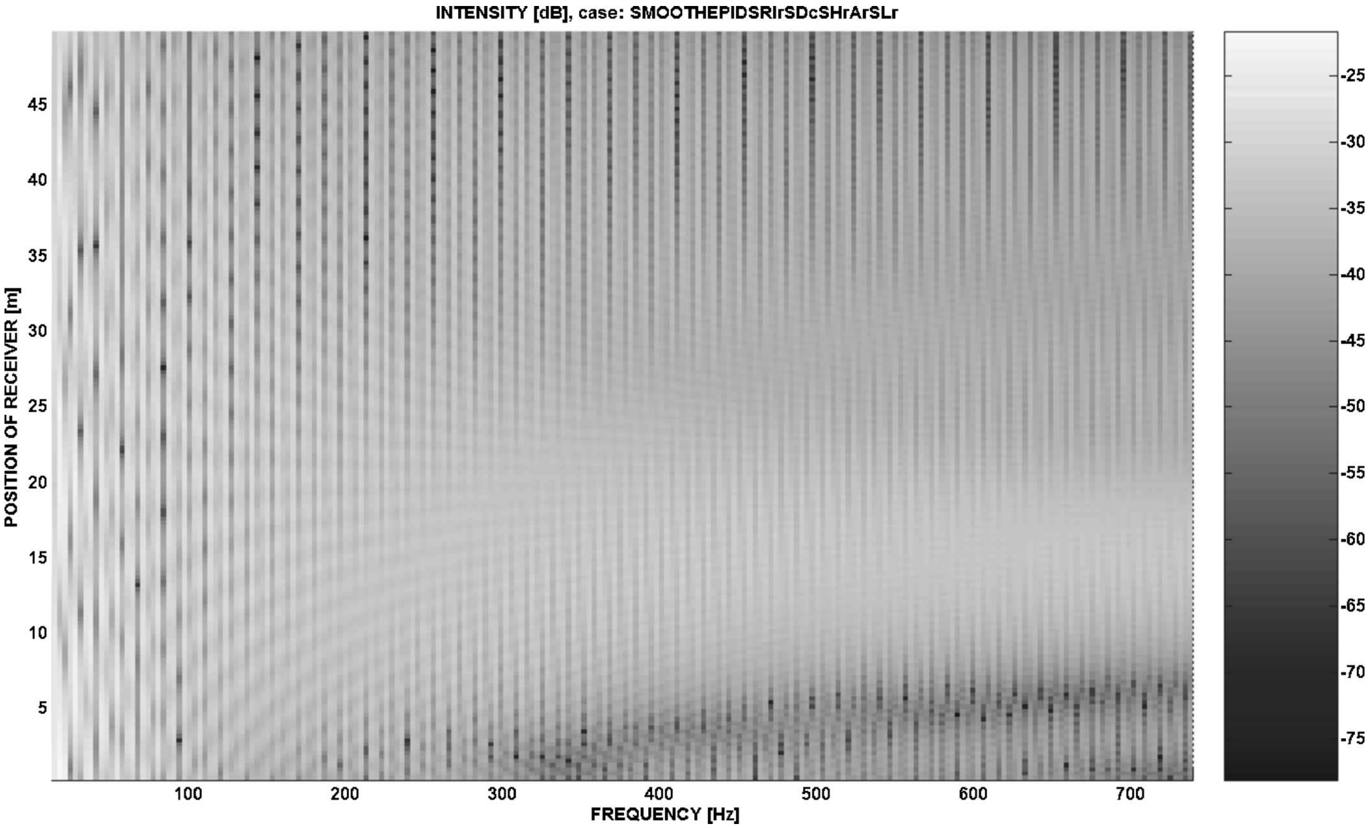

FIG. 5. Similar to Fig. 4, but with incorporation of reflections on the foreground. Reflections on the foreground are responsible for a better distribution of sound throughout the theatre.

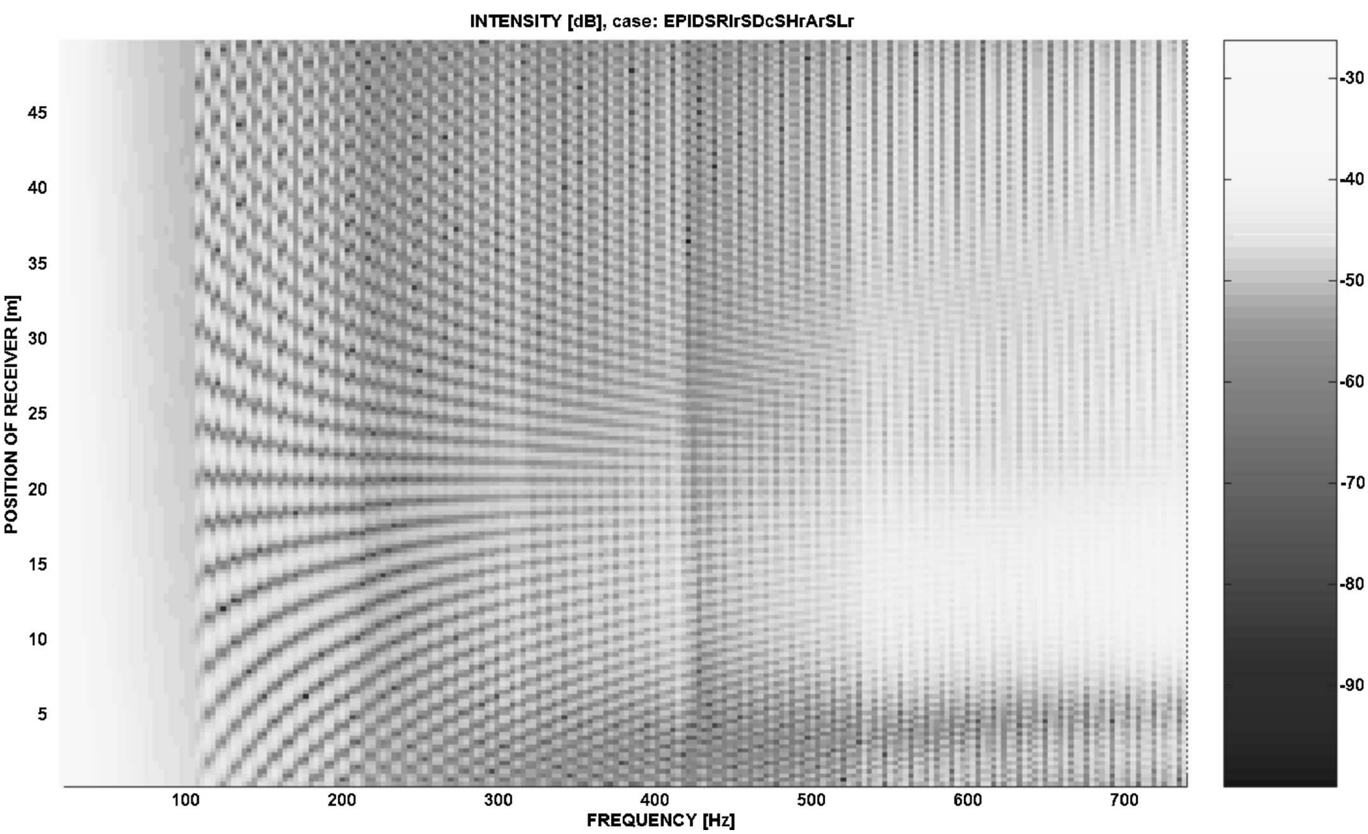

FIG. 6. Calculated intensities, comparable with Fig. 5, but with the seat rows installed. The sound patterns are now influenced by the effect of diffraction. Note that there is a relatively increased amplitude noticeable for high frequencies, whereas the overall sound intensity is lower than in the case without seat rows.

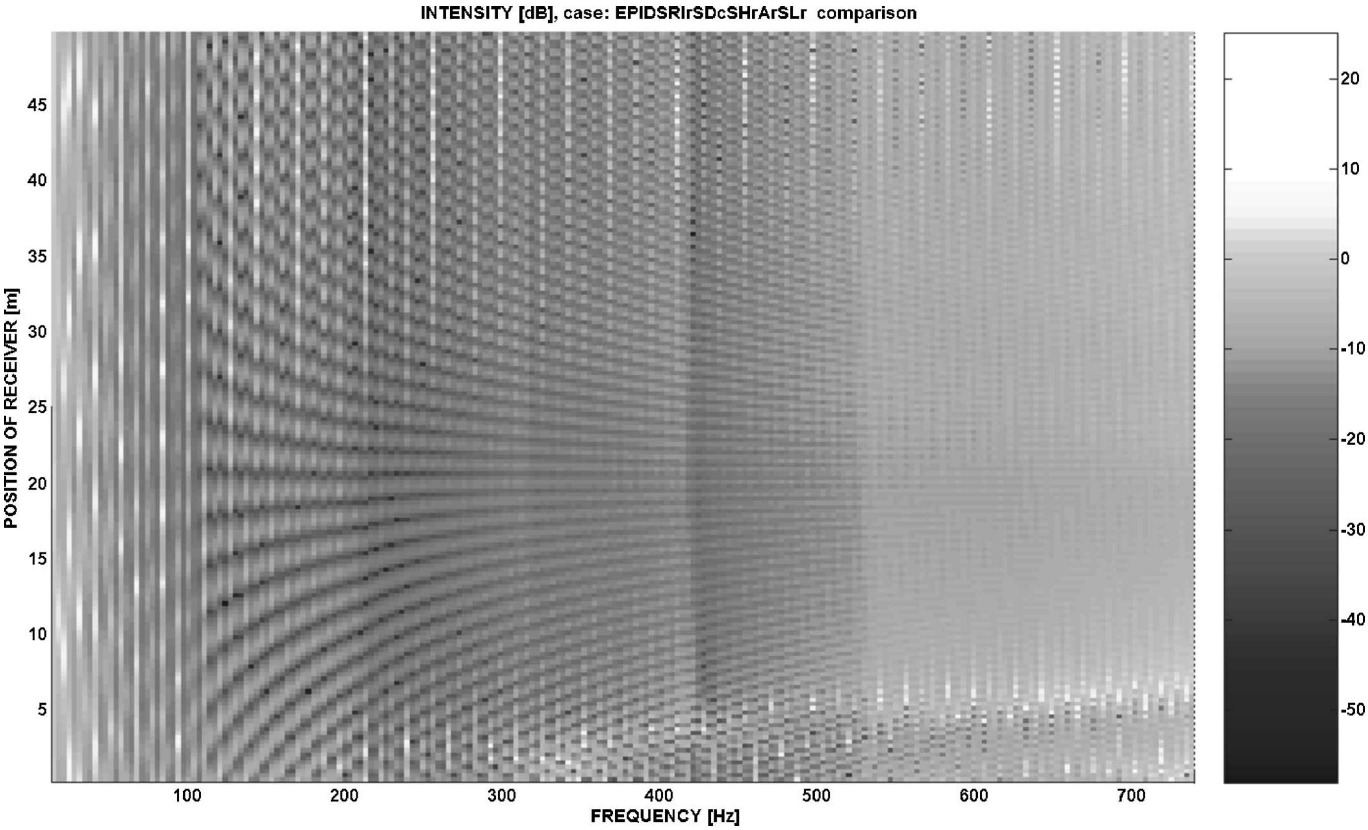

FIG. 7. Comparison of Fig. 6 with Fig. 5, highlighting the effect of diffraction due to the installation of seat rows. At most positions and for most frequencies, the intensity is diminished. However for frequencies beyond 530 Hz, one can see a relatively increased intensity. This is due to the filter effect caused by the seat rows.

Note that the intensities are slightly lower than for the case without seat rows. In other words, the installation of seat rows has a negative effect on the overall intensity of sound throughout the theater.

As a matter of fact, the results are not really simple to interpret because they show the cumulative effect caused by the installed seat rows, caused by reflections on the foreground and caused by the effect of the slope of the theater.

In order to highlight the particular effect of the seat rows, which is the main purpose of this paper, it is necessary to subtract Fig. 6 from Fig. 5. The result is shown in Fig. 7.

Because of the complexity of the diffraction phenomenon, the results are not really "smooth." Still there are certain tendencies visible. First, the relative intensities are almost everywhere negative. This means that the presence of stairs has a "damping effect" due to scattering in multiple directions. Nevertheless, an overall drop of intensity is not dramatic as the human ear is capable of adjusting its sensitivity. What is more important is the fact that frequencies beyond 530 Hz are less damped than frequencies between 50 and 530 Hz. Therefore there is a relative amplification of high frequencies. There is also a dependency of the position in the theater on the observable intensity, but this is mainly caused by the slope and not really by the seat rows, as can be seen in Fig. 5.

Note that reflections on the foreground are also very important for the real theater of Epidaurus. This can be clearly seen in Fig. 8, which is comparable to Fig. 7, except that reflections on the foreground are neglected.

There are high intensity bands appearing from down under to right up, which are merely due to interference effects due to straight sound and zero order diffracted sound that reaches the listener after diffraction. These bands correspond to low physical intensities and are very distinct when the theater contains no seats. In other words the existence of a reflective foreground results in a better distribution of sound throughout the theater and this redistribution is further improved, in addition to the filtering effect in favor of high frequencies, by the presence of seat rows.

Further results (left out of the paper) have revealed the influence of the seat row periodicity on the acoustics. For Aphrodisias, with a periodicity of 0.736 m, the relatively amplified frequencies are higher than 600 Hz. For Pergamon, with a periodicity of 1.657 m the relatively amplified frequencies begin around 300 Hz. In other words, the periodicity of the seat rows influences the band of amplified frequencies: the smaller the periodicity, the higher the amplified frequency band.

Additional results (equally left out of the paper) show that patterns appearing at a certain height on the slope of the theater shift to higher positions if the source is placed higher.

We have also studied the effect on the acoustics of the distance between the source and the first seat row (the "prohedriai"). Apart from an increased overall intensity when the source is positioned closer to the seat rows, we did not detect any spectacular effects except that reflections on the foreground become less important for a source closer to the seat rows, therefore destroying the positive effect of a better dis-

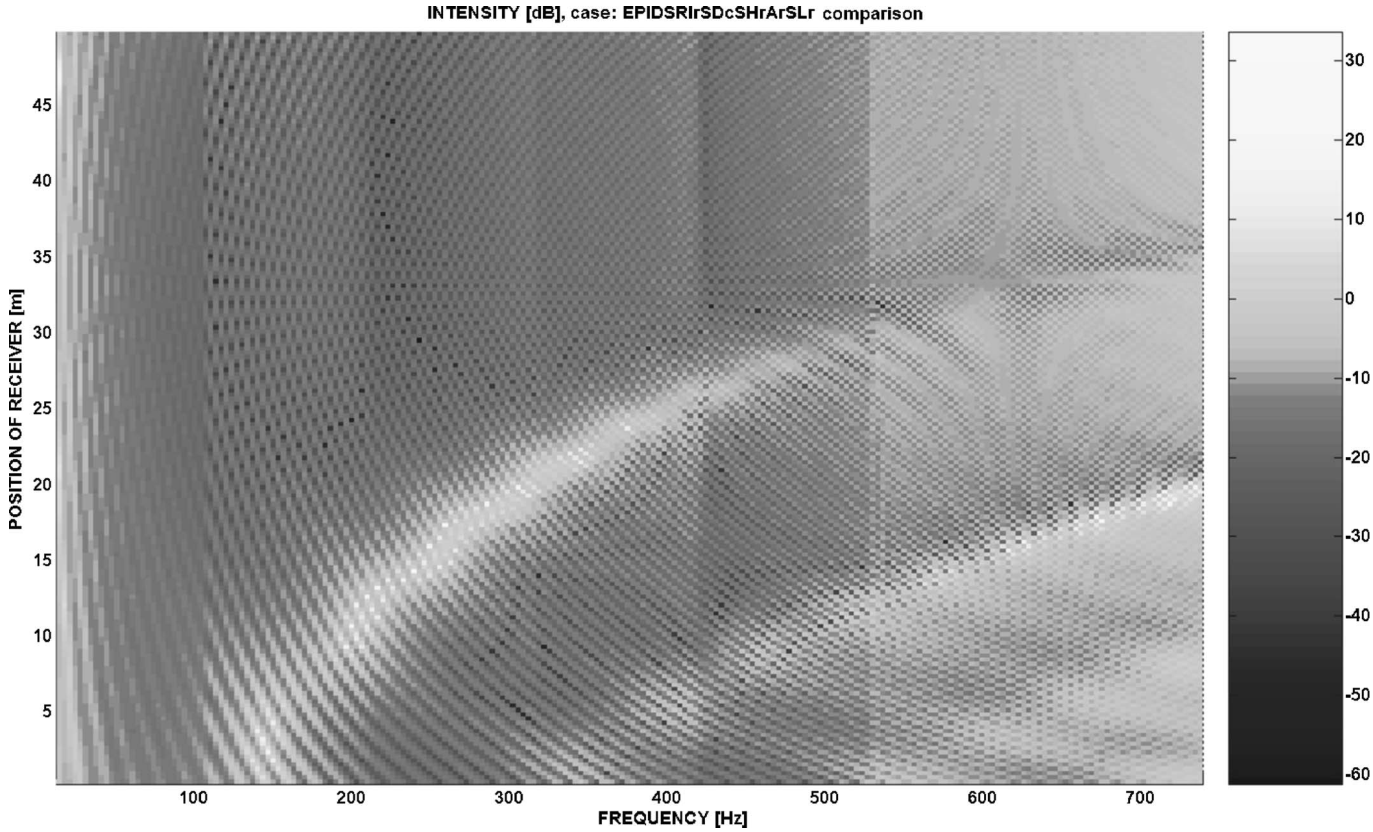

FIG. 8. Comparable to Fig. 7, but in the case of the absence of a reflective foreground. Frequencies above 530 Hz are still favored by the filtering effect, but there appears a position dependent intensity which is caused by the slope, just as in Fig. 4, and is only partly annihilated by the seat rows. This is mainly due to the interaction of uninterrupted sound beams and zero-order diffracted (i.e., undiffracted) sound beams reflected from the cavea.

tribution of sound throughout the theater. Still diffraction of sound on the seat rows makes the effect less dramatic.

We have also studied the influence of the slope of the theater on the acoustics. This effect is very important for a smooth theater without reflections on the foreground. The effects are still noticeable in the case of installed seat rows and reflections on the foreground, but it is less outspoken. The slope does not really influence the frequency values where the amplified frequency band appears.

The previous results were all for summer. Another aspect that we have studied is the influence of the season on the acoustics of Epidaurus. The season has an influence on both the sound velocity in air and the density of air. The differences in the limestone are negligible. We found that there was no significant difference between the acoustics in summer and the acoustics in winter.

VI. THE PHYSICAL ORIGIN OF THE HIGH PASS FILTER EFFECT

For low frequencies, the seat rows do not really diffract sound, which means that there is no big difference compared with a smooth slope. For higher frequencies, diffraction plays a role and higher order reflected sound is generated, causing sound to be distributed in different directions upon reflection into the air and upon transmission into the bulk of the theater's slope. Figures 9 and 10 show the intensity of the isolated reflected diffraction orders "-1" and "-2," respectively.

There is a "vertical raster" added to the figure that indicates the transition from evanescent waves to propagating bulk waves; for frequencies below the raster, sound is evanescent and is stuck to the slope of the theater, for frequencies passing the raster, sound is really propagating in space and is observable. Note that there are negative first-order diffracted waves observable at frequencies above 200 Hz, but that their intensity is really small (−15 dB and much less). The second negative order diffracted waves appear beyond around 450 Hz and their intensity is higher (−10 dB and higher). These facts result in the following analysis: For low frequencies there is no significant effect caused by the seat rows. For higher frequencies the reflected sound is distorted by the diffraction effect resulting in a distribution of the sound energy in many directions and actually causing a drop in the measured sound intensities for the audience. For frequencies in between 100 and 500 Hz, there is a physical influence of the negative first-order diffracted waves on the acoustics of the theater. The amplitude of these first-order waves is very small and therefore it mainly causes a distortion of the sound field and diminishes the observed intensities. For frequencies beyond 500 Hz, the negative second-order waves become important and they do have a significant intensity. These negative second-order waves actually consist of backscattered sound; for a given listener somewhere on the cavea of the theater, they consist of sound that has passed the listener and is backreflected toward this person. Because the accompanied intensity is considerable, it results in an

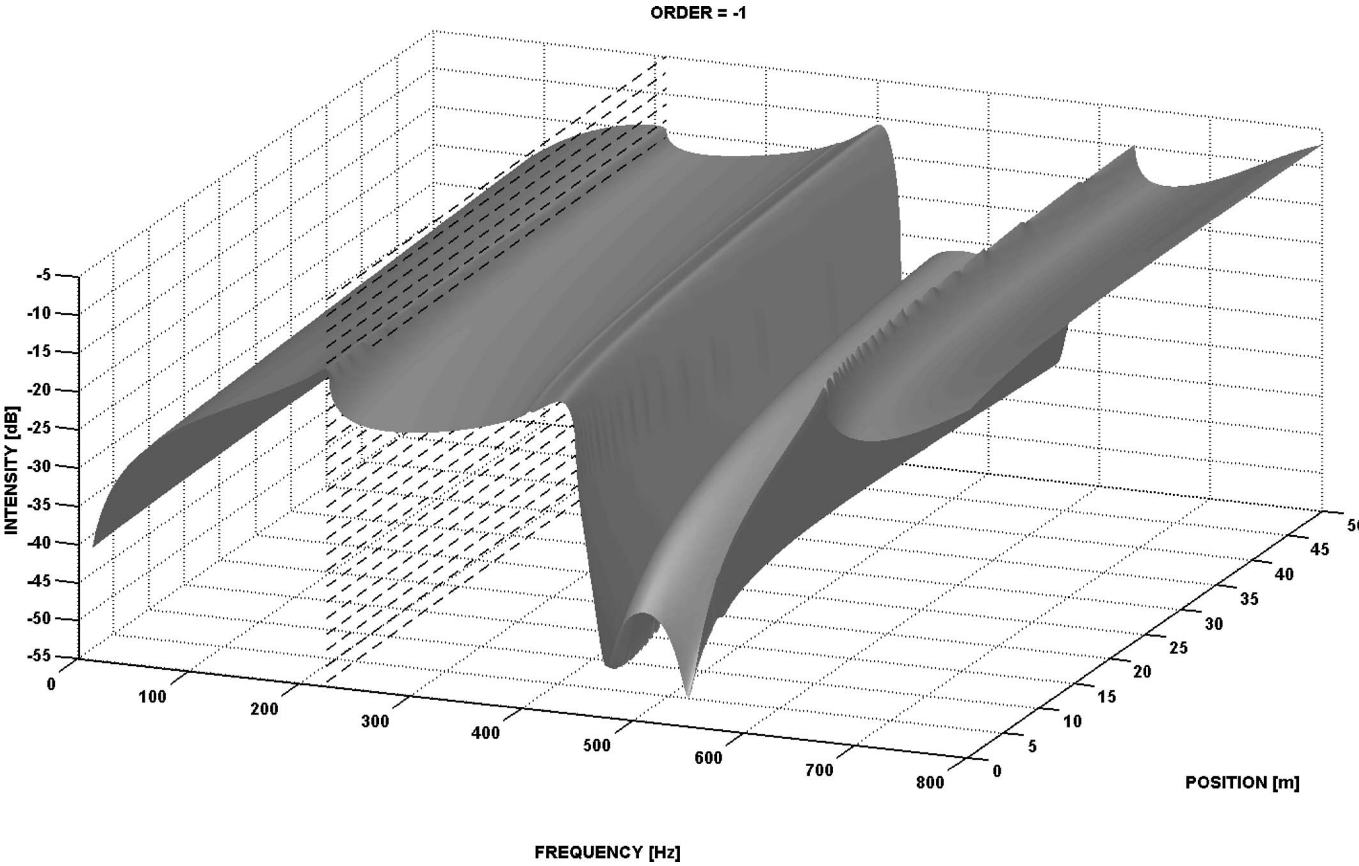

FIG. 9. The diffraction spectrum as a function of frequency and position along the "slope" of the cavea, of the -1 order diffracted sound waves. The raster at approximately 200 Hz indicates the transition between evanescent sound and propagating sound. Note that the amplitudes at frequencies beyond 200 Hz are very small: −15 dB and much less.

increased observed intensity. Contrary to lower frequencies, here the diffraction effect plays a constructive effect.

Besides negative diffraction orders, there are of course also positive diffraction orders involved at Epidaurus. These orders, however, are evanescent throughout the entire considered frequency interval and are therefore never observable by the audience.

VII. CONSEQUENCES OF NUMERICAL RESULTS FOR EPIDAURUS

We have shown in Sec. VI that the most important effect caused by the seat rows at Epidaurus is the effect of relative amplification of a frequency band above 530 Hz. In this section, we discuss the consequences of this effect for the acoustics of the theater.

Izenour[3] already pointed out that background noise is very important for the acoustics of a theater. Background noise is extremely important in a modern motorized society, still at the old Epidaurus there were many visitors that must have caused noise too. Furthermore there is also wind, typically[30,31] up to 500 Hz, rustling trees, etc. Most of the noise produced in and around the theater was probably low frequency noise and even if high frequency noise was produced to some extent, it would have been filtered out by the fact that low frequency noise always spans much further in open air than high frequency noise. The presented calculations indicate that a high frequency band is favored at the expense of lower frequencies. This is true for sound produced at the location of the speaker. Sound coming from other directions will be influenced differently. Nevertheless, we have shown that the position of the sound source and its height has no significant influence on the properties of the amplified frequency band. This means that the conclusions hold for noise coming from any direction.

Still, a reduction of the lower frequencies does not only filter out low frequency noise, but it also filters out the fundamental tones of the human voice (85–155 Hz for men, 165–255 Hz for women). This is not dramatic as the human nerve system and brain are able to reconstruct this fundamental tone, by means of the available high frequency information; this is the phenomenon of virtual pitch in the case of a missing fundamental tone.[32–36] As a matter of fact, virtual pitch is the basic effect behind the creation of the illusion of bass in small radios, miniature woofers, and in telephones.[37] In other words the seat rows of the theater filter out low frequency noise which has a positive influence on the clarity of a speaker throughout the theater, despite the fact that the lower tones of the human voice are filtered out as well.

VIII. COMPARISON WITH OTHER CLASSICAL THEATERS

Table IV shows the physical parameters of different ancient theaters.

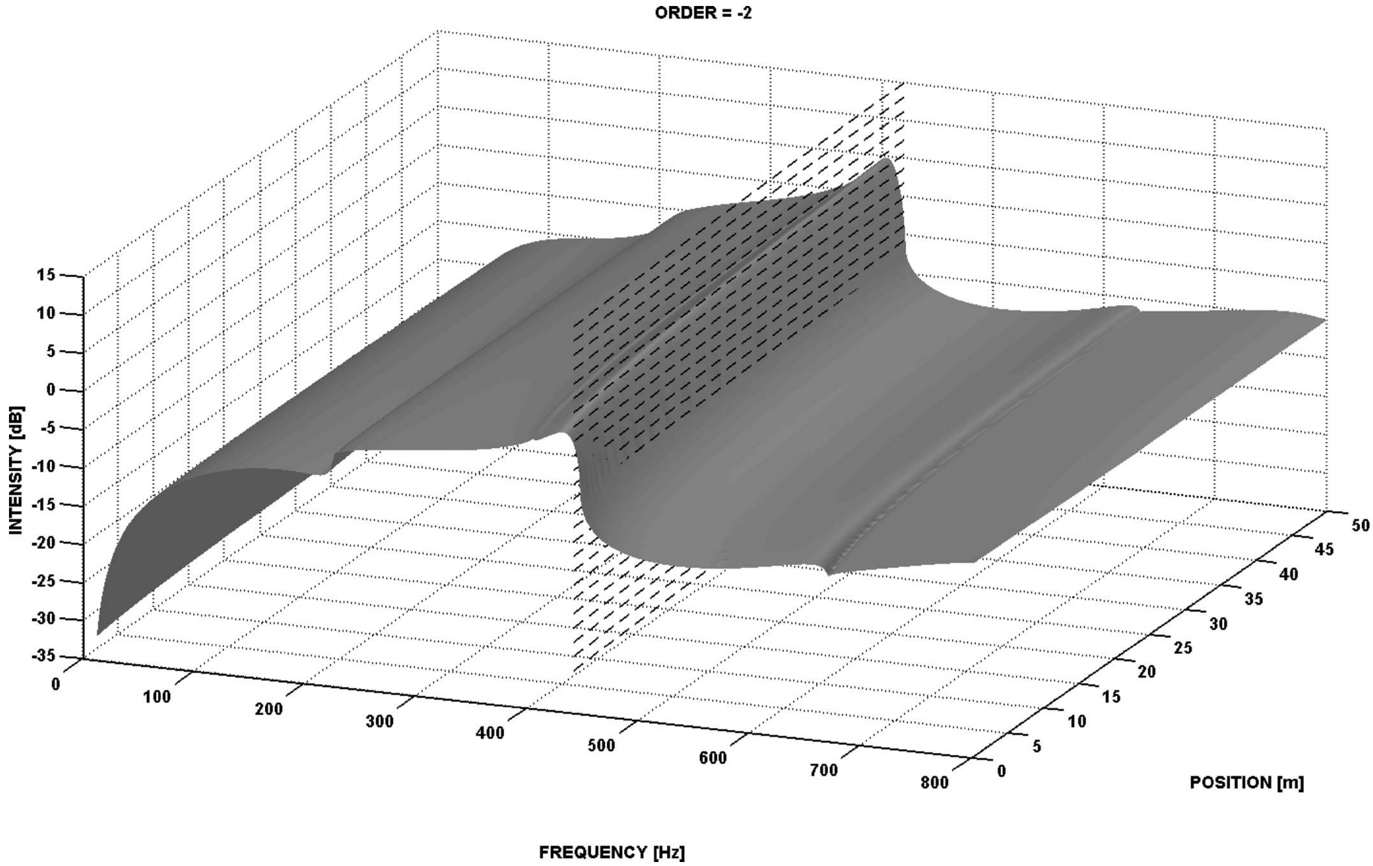

FIG. 10. Comparable to Fig. 9, but for the -2 order diffracted sound. The raster is now situated at approximately 450 Hz. The amplitude for frequencies beyond 450 Hz are −10 dB or higher. It is this -2 order diffracted sound that is responsible for the filter effect and for favoring frequencies beyond 500 Hz for the audience.

Note that most theaters, apart from Pergamon, have a seat row periodicity that is comparable to Epidaurus. The slope values are more scattered. The discussion of our obtained numerical results shows that the periodicity is the key factor for the filtering effect of the stairs. Within that scope, it is not surprising that most theaters copy Epidaurus' seat rows.

TABLE IV. Properties of classical theaters.

Theater	Dated	Location	Style	Λ (m)	α (deg)
Epidaurus[a]	300 B.C.	Greece	Hellenistic	0.831	26.6
Aphrodisias[b]	300 B.C.	Turkey	Hellenistic	0.736	31.1
Aspendos[b]	161–180 A.D.	Turkey	Roman	0.788	33.1
Dionysus[c] (Athens)	400–300 B.C.	Greece	Greek	0.829	23.5
Ostia Antica[b]	19–12 B.C.	Italy	Roman	0.762	22.1
Pergamon[b]	197–159 B.C.	Turkey	Roman	1.657	62.7
Pompeii[d] (Odium)	80 B.C.	Italy	Roman	0.805	32.3
Priene[b]	330 B.C.	Turkey	Hellenistic	0.772	31.2
Side[b]	?	Turkey	Greek	0.749	34.1

[a]See Ref. 1.
[b]See Ref. 38.
[c]See Ref. 39.
[d]See Ref. 40.

Still, the fact that the acoustics of Epidaurus is much more renowned than the acoustics of the other theaters is probably because of the fact that Epidaurus has been renowned from the very beginning (historical reason) and that it has been preserved so well (conservational reason).

Dionysus is the theater whose dimensions best resemble the dimensions of Epidaurus, but is in a much worse condition and therefore it will never be really possible to compare the acoustics of both theaters experimentally.

IX. CONCLUDING REMARKS

It is shown that reflections on the foreground of the theater result in a better distribution of sound throughout the cavea so that all positions become acoustically similar to one another. The installation of seat rows on a smooth cavea generates diffraction effects that change the acoustic properties of the theater.

The intensity observed by the audience will be lower than in the case of a smooth cavea. This is not dramatic because the human ear is capable of adapting its sensitivity. It is more important that the damping effect is frequency dependent: the seat rows act like a filter. For frequencies beyond a certain threshold, second-order diffracted sound plays an important role and causes sound to be backscattered from the cavea to the audience making the audience receive sound from the front, but also backscattered sound from behind. This has a positive outcome on the reception of sound.

For frequencies below the threshold (mostly noise), the effect of backscattering is less important and is to a great extent filtered out of the observed sound. The threshold frequency of the filtering effect is mainly determined by the periodicity of the seat rows in the cavea of the theater. For Epidaurus this threshold is around 500 Hz, which is usually the upper limit for wind noise.[30,31]

The slope of the cavea does not really influence the frequency values where the amplified frequency band appears and there is no significant difference between the acoustics in summer and the acoustics in winter.

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